Nonlinear Oscillators in Space Physics

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Abstract. We discuss dynamical systems that produce an oscillation without an external time dependent source. Numerical results are presented for nonlinear oscillators in the Earth's atmosphere, foremost the quasi-biennial oscillation (QBO). These fluid dynamical oscillators, like the solar dynamo, have in common that one of the variables in a governing equation is strongly nonlinear and that the nonlinearity, to first order, has a particular form, of 3rd or odd power. It is shown that this form of nonlinearity can produce the fundamental frequency of the internal oscillation, which has a period that is favored by the dynamical condition of the fluid. The fundamental frequency maintains the oscillation, with no energy input to the system at that particular frequency. Nonlinearities of 2nd or even power could not maintain the oscillation.

Keywords: Wave-driven quasi-biennial oscillation; Intra-seasonal bimonthly oscillation; Semidiurnal pseudo tide; 22-year solar dynamo.

1. Introduction

By way of introduction to this tutorial review on nonlinear oscillators (NLO), we present as an example the mechanism of the pendulum clock, as illustrated in Fig. 1a. With sufficient energy from the suspended weights to overcome friction, the escapement mechanism provides the impulse/nonlinearity that keeps the pendulum oscillating[*]. The resonance frequency for the pendulum, determined by the length of the pendulum, is produced without an external time dependent source. The external source is the weight, a steady force, pulling on the drive chain of the clock, which has no power at any frequency.

An external time dependent source is required to generate an oscillation with a linear system of n equations and n unknowns, $L(n,n) \cdot u(n) = s(n)$. Here L is the square matrix of linear terms that account for dissipation, u the unknown column vector, and s the source vector. With $u(n) = L^{-1}(n,n) \cdot s(n)$, a linear finite solution with nonzero u, requires that s is also nonzero. It is understood that finite differencing produces the matrix elements for L to obtain a solution for a system of differential equations.

Considering linear oscillators (LO), an example is the radio tuner, with inductance, capacitance, and resistance, which picks up or filters out a particular frequency amongst the incoming wave spectrum, as illustrated in Fig. 1b. Unlike the nonlinear clock mechanism, the radio receiver does not produce the frequency on its own. In fluid dynamics, gravity waves are well represented by linear oscillations. Figure 1c (Mayr and Volland, 1976) shows N_2 density variations at 250 km computed for spherical harmonics, P_n^0, with n = 10, 12, 14, plotted versus frequency. Maxima develop at distinct frequencies that increase in proportion to the wave number, n, consistent with the dispersion relationship for gravity waves. The results were obtained from a model that solves the linearized equations, allowing for dissipation due to viscosity and heat conduction. Joule heating, with variable frequency, was the energy source that generated

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[*] Encyclopedia Britannica (1999): "In a pendulum clock, an escape wheel is allowed to rotate through the pitch of one tooth for each double swing of the pendulum, and to transmit an impulse to keep it swinging."

the oscillations. Other examples of LO are the diurnal and semidiurnal tidal oscillations, which are generated by solar heating with periods of 24 and 12 hours, respectively.

The LO and NLO have in common, that they experience dissipation and accordingly require energy to generate the oscillations. Without friction, the pendulum in a fictitious mechanical clock, once initiated, would continue oscillating without the energy supplied by the weights. And the fictitious radio tuner without resistance, and gravity waves without dissipation, once excited, would continue oscillating without the external time dependent excitation sources.

In the following, we shall discuss examples of NLO in fluid dynamics, highlighting the nonlinearities involved, and the processes that produce the periodicities. We shall present numerical results for the Earth atmosphere, foremost the quasi-biennial oscillation (QBO), which has been studied extensively to provide understanding (e.g., Mayr et al., 2010). In light of these results, we briefly discuss the nonlinearity that is involved in generating the 22-year oscillation of the magneto-hydrodynamic (MHD) solar dynamo.

2. Nonlinear Oscillators in the Earth Atmosphere

2.1. Quasi-biennial Oscillation (QBO)

In the zonal-mean (m = 0) zonal circulation of the lower stratosphere at low latitudes, the quasi-biennial oscillation (QBO) dominates with periods between 21 and 32 months. Lindzen and Holton (1968), and Holton and Lindzen (1972) demonstrated that the QBO, in principal, can be generated with planetary waves (PW), and their theory was confirmed in numerous modeling studies. More recent studies with realistic PW have led to the conclusion that small-scale gravity waves (GW) are more important for the QBO (e.g., Hitchman and Leovy, 1988). The GWs generally need to be parameterized, and in several models this wave source has been successfully applied, reproducing the QBO (e.g., Mengel et al., 1995; Dunkerton, 1997; Lawrence, 2001; Giorgetta et al., 2002).

Lindzen and Holton (1968) discussed the unique dynamical conditions that produce the QBO. At the equator, where the Coriolis force and the meridional winds vanish, the wave-generated zonal winds are dissipated only by viscosity. Away from the equator, the meridional circulation increasingly comes into play to dissipate the zonal flow oscillation, in part through radiative cooling. This property is displayed in model simulations with latitude independent wave source (e.g., Mengel et al., 1995), which still produce QBO zonal winds that peak at the equator, as observed.

In their seminal paper of the QBO, Lindzen and Holton (1968) explained the oscillation as being driven by the semi-annual oscillation (SAO) above 40 km. Holton and Lindzen (1972) subsequently concluded that the seasonal variations of the SAO were important but not essential for generating the QBO -- and Mayr et al. (1998a) confirmed that in a study with the Numerical Spectral Model (NSM).

The numerical design of the NSM was introduced by Chan et al. (1994). Starting with Mengel et al. (1995) and Mayr et al. (1997), the model has been run with the Doppler spread parameterization (DSP) of Hines (1997a, b) that has been employed also in a number of other global-scale models (e.g., Manzini et al., 1997; Akmaev, 2001a, b). In the NSM, the DSP produces the GW momentum source and associated eddy viscosity,

which are of central importance for the numerical results discussed. From that model, we present in Fig. 2 the computed zonal winds (Mayr et al., 1998a). Applying a time invariant GW source, the computer solution was produced for perpetual equinox, without time dependent solar heating, and it shows QBO-like oscillations generated in the lower stratosphere. Without an external time dependent source, the excitation mechanism for the oscillation cannot be linear, as is readily shown in Section 1. The QBO must be understood to be a nonlinear oscillator (NLO) -- and we ask what the nature is of the nonlinear process involved.

The nonlinearity is displayed in Fig. 3, which shows snapshots of the zonal winds and the accompanying GW momentum source. (For comparison with the winds, the source was divided by the height-dependent mass density and eddy viscosity.) Discussed in Mayr et al. (1998a, b), the positive nonlinear feedback associated with critical level of wave absorption (Booker and Bretherton, 1967; Hines and Reddy, 1967) produces a GW momentum source (MS) with sharp peaks near zonal wind shears, dU/dr, in short DU. The MS in Fig. 3 is clearly a nonlinear function of the velocity gradient, and we ask what the analytical form is that can qualitatively describe the nonlinearity. If the nonlinearity were of 2nd or even power, e.g., DU^2 or DU^4, the MS would point into the positive direction, irrespective of the positive or negative sign of the vertical gradient, DU, which is obviously not the case. The MS is instead pointing in the direction of DU, which means that the nonlinear part of the MS can be approximately described having an analytical form of 3rd or odd power, e.g., DU^3 or DU^5. Considering the wind oscillation of the QBO, varying with altitude, r, and varying with time (Fig. 2a) or oscillation period (Fig. 2b) with frequency, ω, the nonlinear part of the MS, to first order, can be written in the approximate form $\propto [dU(\omega,r)/dr]^3$. In frequency space, and with abbreviated complex notation, such a nonlinear source in the form $\propto [Exp(i\omega t)]^{3*}$ produces the term with $Exp[i(\omega + \omega + \omega)t]$ for the higher order frequency, 3ω. Moreover and of crucial importance, such a nonlinear source of 3rd (or odd) power also generates the term with $Exp[i(\omega + \omega - \omega)t]$ for the fundamental frequency, ω, that maintains the oscillation. [Nonlinearities of 2nd or even power, e.g., $[Exp(i\omega t)]^2$, would produce $Exp[i(\omega + \omega)t]$ and $Exp[i(\omega - \omega)t]$, without the fundamental frequency to maintain the oscillation.]

Having identified the nonlinear mechanism that generates the QBO, we turn to the process that dissipates the oscillation. As emphasized above, the QBO at the equator is dissipated by the eddy viscosity/diffusivity, K. Figure 4a shows that K increases exponentially as the GW amplitude grows with height. The corresponding time constant is given by $\tau = \lambda^2/2\pi K$, with λ the vertical wavelength of the oscillation, and it produces periods that decrease with altitude as shown in Fig. 4b. Considering the importance of vertical transport processes, the large height variations of τ (and K) are not translated into corresponding variations of the zonal wind periodicity. In qualitative agreement with Fig. 4b, nonetheless, the oscillation pattern in Fig. 2 varies with periods that decrease with altitude from about 2 years at 35 km to values around 7 months at 50 km.

The connection between dissipation and oscillation period is further illustrated in a computer experiment presented in Fig. 7 of Mayr et al. (1998a). With identical wave source, the zonal winds were derived for eddy diffusion rates a factor of two smaller. Compared with the zonal winds in Fig. 2, the oscillations then have periods about a factor of two longer, and the amplitudes are larger by about the same factor.

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[*] $Cos(\omega t)^3 = Cos(3\omega t)/4 + 3Cos(\omega t)/4$

We have shown that without external time dependent solar forcing, and applying a constant tropospheric wave flux, GW interactions can generate stratospheric QBO like oscillations. Due to the nonlinear positive feedback associated with critical level absorption, which applies also to planetary wave interactions, the GW momentum source is strongly nonlinear, of odd power. It is shown that this form of nonlinearity can produce the fundamental frequency that maintains the oscillation. The eddy viscosity mainly dissipates the oscillation and controls its amplitude. And the eddy viscosity produces time constants around 2 years for the periodicity, hence the quasi-biennial oscillation.

2.2. Intra-seasonal Bimonthly Oscillation (BMO)

Zonal-mean intra-seasonal oscillations with periods around 2 months have been seen in ground based measurements of winds and gravity wave activity observed at equatorial latitudes in the upper mesosphere and lower thermosphere (e.g., Eckermann and Vincent, 1994). Similar oscillations have been seen in satellite wind measurements (e.g., Liebermann, 1998; Huang and Reber, 2003), and Huang et al. (2006) inferred such variations from spacecraft temperature data. The oscillations have also been observed in polar mesospheric clouds (Bailey et al., 2005).

The Numerical Spectral Model (NSM) generates intra-seasonal oscillations (Mayr et al., 2003), and numerical experiments with the 2D version of the model show that they are not produced when the GWs propagating north/south are turned off. In Fig. 5a we present a computer solution from that paper, which is produced only with the meridional GW momentum source. Pronounced and regular oscillations are generated with a period of about 2 months, which propagate down through the atmosphere. Except for the shorter period and smaller wind velocities, the bimonthly oscillation (BMO) resembles the quasi-biennial oscillation (QBO) above discussed.

The similarity between this BMO and the QBO is furthermore evident in Fig. 5b, where a snapshot of the meridional winds is presented together with the associated momentum source. (Since the relative variations are of interest, normalization factors are applied so that the quantities can be resolved throughout the atmosphere.) As seen for the zonal GW source of the QBO in Fig. 3, the momentum source in Fig. 5b is sharply peaked near meridional wind shears, in particular at altitudes above 50 km. Discussed in Section 2.1., this impulsive momentum source can be viewed approximately as a nonlinear function of the velocity field, which has the character, $[dU/dr]^3$, of odd power. Such a non-linearity can generate an oscillation without external time-dependent forcing. Like the QBO, the meridional BMO can be understood as a nonlinear oscillator.

In the case of the QBO, the time constant is determined by the eddy viscosity that dissipates the zonal flow near the equator. The present BMO is also dissipated by viscosity. But unlike the rotational zonal winds of the QBO, the divergent wave-driven meridional winds of the BMO generate dynamical heating and cooling. The resulting pressure variations thus counteract, and dampen, the winds to produce a shorter dissipative time constant. Discussed in Mayr et al. (2003), this additional thermodynamic feedback could explain why the period of the meridional wind oscillation (Fig. 5a) is much shorter than that of the QBO zonal winds (Fig. 2). In both cases, the dissipation rates determine the oscillation periods.

2.3. Semidiurnal Pseudo Tide

In a recent study with the Numerical Spectral Model (NSM), Talaat and Mayr (2011) showed that the model generates, at high latitudes, semidiurnal (12-hour) oscillations without time-dependent solar excitation of the tides. These pseudo tides attain amplitudes that are, at times, as large as those of the regular non-migrating tides produced with standard solar heating, as shown in Fig. 6.

With Hines' Doppler spread parameterization (DSP) applied in the NSM, the GW momentum source amplifies the tides and planetary waves through critical level absorption (e.g., Mayr et al., 2011). As shown above, the positive nonlinear feedback in that process produces an impulsive wave source (Fig. 3) that generates the quasi-biennial oscillation (QBO) without external time dependent forcing. For the nonlinear QBO, the periodicity is controlled by the eddy viscosity, which produces a time constant of about 2 years. For the semidiurnal pseudo tide in the present case, Talaat and Mayr proposed that the Coriolis force, $\propto 2\Omega \sin(\theta)$ (θ, latitude), would favour the excitation of a 2Ω (12-hour) oscillation at high latitudes. Like the QBO and clock, the semidiurnal pseudo tide takes on the character of a nonlinear oscillator, and it is produced without external time dependent solar heating.

3. Babcock-Leighton Solar Dynamo

Charbonneau (2005) reviewed the extensive literature on magneto hydrodynamic (MHD) dynamo models of the 22-year solar cycle and related observations. Pars pro toto, we refer to the model of Dikpati and Charbonneau (1999), which is built, like so many other models, on the early developments of Babcock (1961) and Leighton (1969). Dikpati and Charbonneau generate the solar oscillation with a kinematic 2D zonal-mean magnetic field model. Applying the observed solar differential rotation inferred from helioseismology (e.g., Scherrer et al., 1995), taken time invariant, the poloidal magnetic field is wound up to produce a toroidal field (Ω-effect) in the tachocline below the convective envelope. The buoyant toroidal field emerges at the top of the convection region to form bipolar magnetic regions (BMRs), usually containing sunspots. In that process, the toroidal field in rising flux tubes is twisted by the Coriolis force, thereby regenerating the poloidal field. This has been labeled the α-effect, since the pattern closely matches the Greek symbol.

Dikpati and Charbonneau artificially add in their equation for the poloidal magnetic field the source term

$$S(r, \theta; B_\varphi) \propto \left[1 + \left(\frac{B_\varphi(r_c, \theta, t)}{B_o} \right)^2 \right]^{-1} \times [\sin\theta\cos\theta] \times B_\varphi(r_c, \theta, t),$$ (1)

which can be expanded, in the familiar Euler expansion, to yield

$$\propto [\sin\theta\cos\theta] \times \left[B_\varphi - \frac{B_\varphi^3}{B_o^2} + \frac{B_\varphi^5}{B_o^4} + ... \right].$$ (1a)

Here B_φ represents the toroidal magnetic field, (r, θ) are spherical polar coordinates (r_c, for the tachocline region), and the value $B_o = 10^5$ G is chosen. In Eq. (1), the so-called α-

quenching term $\left[1 + \left(B_\varphi(r_c, \theta, t) / B_0\right)^2\right]^{-1}$ has the desirable property of rapidly weakening the poloidal field when the toroidal field exceeds B_0.

The nonlinearity of the source term of Eq. (1), the only nonlinear term in the model, has the property that it is of odd (e.g., 3rd) power, and that the nonlinearity points into the direction of the oscillating B_φ. [A nonlinearity of odd power appears also in the classical dynamo model of Leighton (1969), where the term $\propto |B_\varphi| B_\varphi / B_c$ describes the eruption of flux in the equation for the toroidal field with threshold B_c.] In dynamo models, the nonlinear terms have the character of the momentum source (Fig. 3) that generates the zonal winds of the quasi-biennial oscillation (QBO). For the solar dynamo and the QBO, the nonlinear source terms have the general form S^3, of odd power. In frequency space, and with complex notation, this kind of nonlinearity $\propto [Exp(i\omega t)]^3$ produces $Exp[i(\omega + \omega + \omega)t]$ with 3ω, and $Exp[i(\omega + \omega - \omega)t]$ with the fundamental frequency, ω, that maintains the oscillation. [Even power nonlinearities could not maintain the oscillation.] The odd power nonlinearities of the QBO and solar dynamo have in common that they are pointing into the direction of the respective oscillations. And the same can be said for the escapement mechanism of the clock, which produces an impulse for each swing direction of the pendulum, as illustrated in Fig. 1a.

The QBO and solar dynamo have furthermore in common that the dissipation rates for the oscillations determine the periodicities. In the case of the QBO it is the eddy viscosity; and in the MHD dynamo model of Dikpati and Charbonneau (1999), the chosen magnitude of the meridional circulation mainly controls the period of the solar cycle.

In this discussion of the solar dynamo, we did not touch on the difficulties solar physicists face in predicting solar activity and observing solar magnetism below the photosphere. There is no incontrovertible proof of where the solar dynamo lies, since we can only glimpse at the solar interior with helioseismology and have not seen direct evidence for deep magnetic fields. During the last few years, predictions of the present solar cycle, # 24 (e.g., Schatten, 2005; Svalgaard et al., 2005; Dikpati et al., 2006), have varied over a wide range, from very small to one of the largest (Pesnell, 2007). Although there is virtually no agreed-upon method to predict solar activity, employing the observed precursor polar magnetic field had led to some success.

4. Concluding Remarks

As is readily shown, a linear system cannot produce an oscillation without an external time dependent source. Nonlinearity is required to generate an oscillation on its own, and the mechanical clock is a prime example. Looking at the ratchet mechanism of the mechanical clock (Fig. 1a), one can see that the nonlinearity it generates has directional character, i.e., the impulse sequence is in phase with the swing direction of the pendulum. Analogous to the clock mechanism, the impulsive nonlinearity (Fig. 3) that generates the quasi-biennial oscillation (QBO) is also directional, pointing in the direction of the velocity gradient, having a form proportional to $[dU/dr]^3$, of 3rd or odd power. And in the solar dynamo, the nonlinear source terms of odd power (Eq. 1) are pointing into the direction of the generated magnetic field. As demonstrated in Section 2.1., nonlinear

source terms of this kind produce, besides higher order frequencies, the fundamental frequency itself to maintain the oscillation – in contrast to even power nonlinearities that cannot generate the frequency. In principle, the dynamical properties of nonlinear oscillators in fluid dynamics are well understood.

The oscillators discussed were chosen to be isolated. The clock was not shaken but was fixed, solely powered by the steady force of the gravitational pull. For the QBO, the wave source and solar input were forced to be constant, time independent. In reality, the QBO is being shaken by the variations of the wave source and solar heating, which amplify and modulate the oscillation to produce a complicated climatology under the influence of the semiannual oscillation (SAO) and solar cycle variations, for example (e.g., Mayr et al., 2010). In the case of the solar dynamo, a nonlinear source term is artificially introduced to produce the 22-year oscillation from a numerical solution, which is generated with prescribed time-invariant differential rotation. Some other source terms, with odd nonlinearity, may also produce desirable solar dynamo features. And the solar dynamo might, for example, be shaken by variations of the energy emanating at the base of the convection region, which would produce time dependent convection patterns and corresponding temporal variations of differential rotation and meridional circulation. These could perhaps be responsible for solar activity variations with some known time scales.

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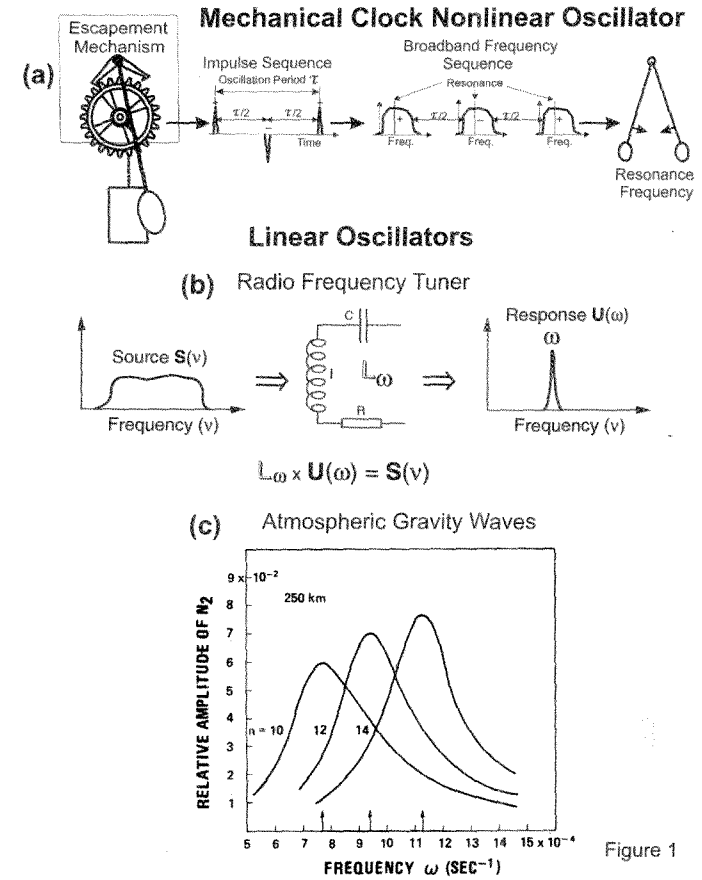mesosphere, J. Atmos. Solar Terr. Phys., accepted for publication.

Figure 1. (a) Example of a nonlinear oscillator (NLO) is the mechanical clock, where the escapement mechanism produces the impulse/nonlinearity that generates the oscillation without external time dependent source. Illustrated are the impulse sequence and broadband frequency sequence, which produce the pendulum oscillation with resonance frequency. Examples of linear oscillators (LO) are: (b) the electric circuit of the radio tuner, which filters out a single frequency (right panel) from the received broadband spectrum (left panel), and (c) gravity wave oscillations with resonance maxima, which are generated by an external time dependent energy source (Mayr and Volland, 1976).

Zonal Winds (m/s)

Perpetual Equinox
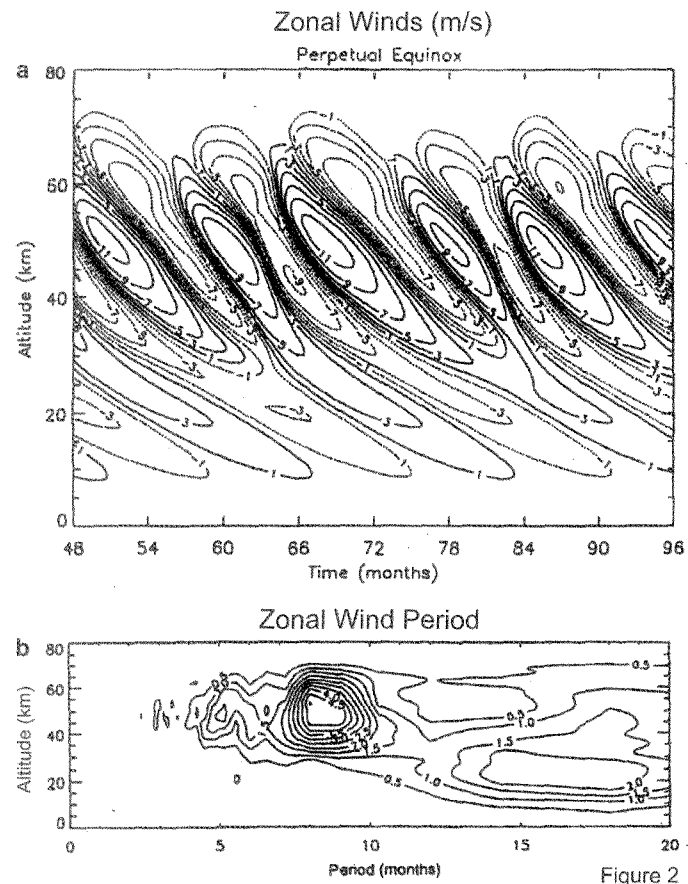

Zonal Wind Period

Figure 2

Figure 2. Shown are equatorial zonal winds (a) and corresponding oscillation periods (b), computed without the influence of time dependent solar heating. A constant gravity wave (GW) flux provides the energy for the momentum source that generates the QBO like oscillation below 35 km (Mayr et al., 1998a).
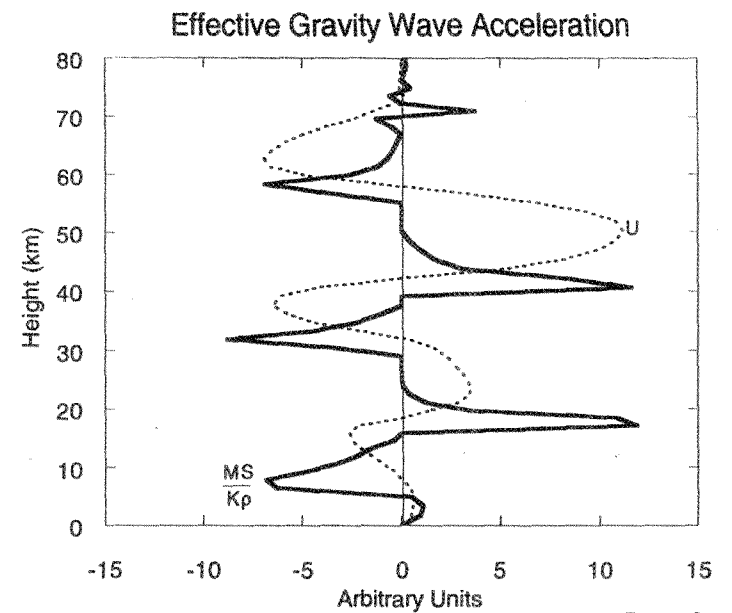
Effective Gravity Wave Acceleration

Figure 3

Figure 3. Snapshots are shown of zonal winds, U, and effective GW momentum source, MS/Kρ, that generates the oscillations (Fig. 2). Note that the peak momentum source occurs near the maximum vertical velocity gradient, and it varies in phase with the gradient. Owing to the positive nonlinear feedback associated with critical level of wave absorption, the momentum source is strongly nonlinear, of 3rd or odd power (Mayr et al., 1999).
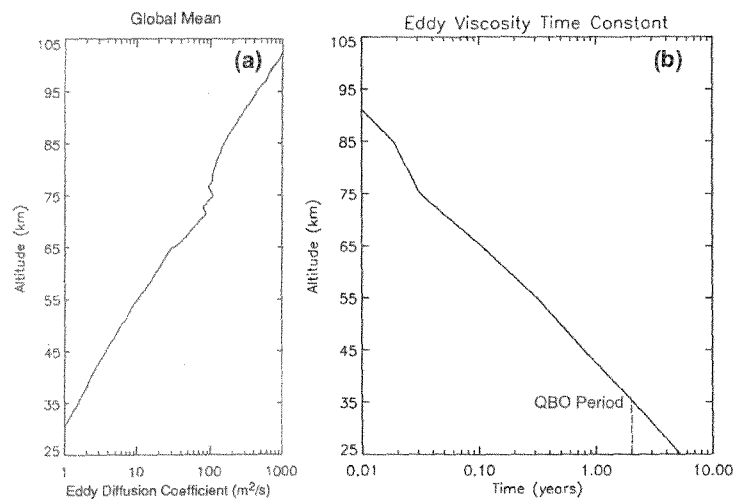
13

14

Figure 4

Figure 4. (a) Altitude variation of eddy diffusivity/viscosity, K, which dissipates the oscillation (Mayr et al., 1997). (b) Eddy viscosity time constant versus altitude. Periods around 2 years define the QBO oscillation.
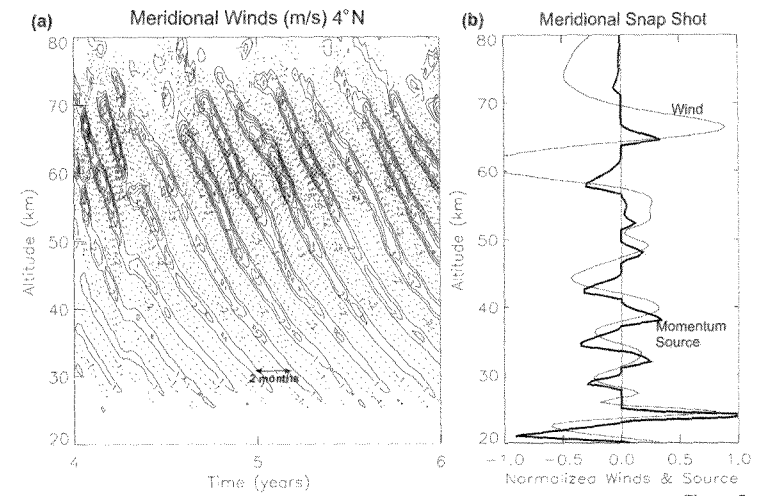

Figure 5

Figure 5. (a) Meridional wind oscillations at 4° latitude, with periods of about 2 months, generated only with meridional GWs propagating north/south. (b) Snapshots are shown of meridional winds, and the associated momentum source produces sharp peaks near vertical wind shears, similar to Fig. 3 (Mayr et al., 2003).
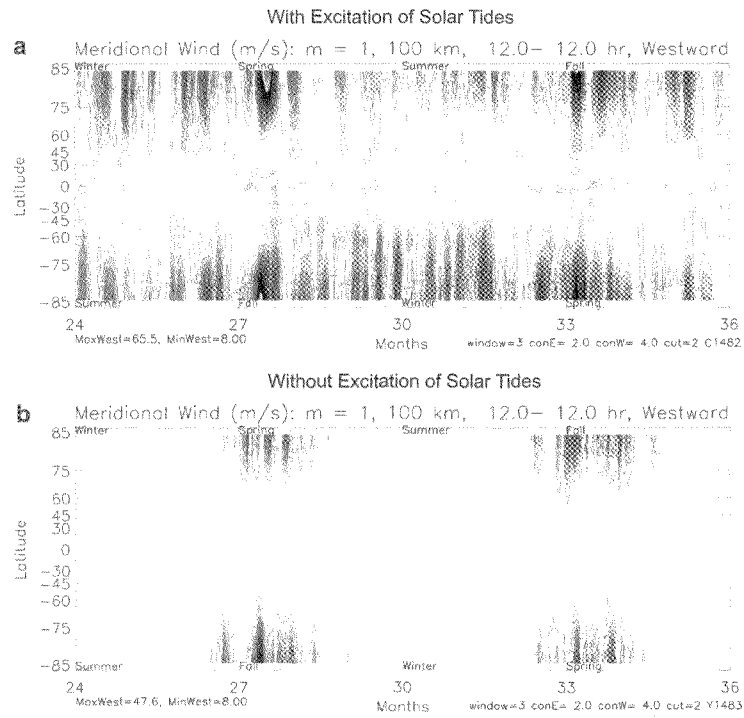
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Figure 6

Figure 6. Seasonal variations of westward propagating 12-hour (semidiurnal) tidal oscillations for m = 1 meridional winds at 100 km, applying a running window of 3 days. Generated with (a) and without (b) solar excitation of the diurnal tides, the oscillations at times have comparable amplitudes (Talaat and Mayr, 2011).